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Usage of LDPC Codes in a Gilbert Channel

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Abstract: *Although low-density parity-check (LDPC) codes in modern communication standards have been extensively studied over a memoryless channel, their burst error correction capacity in channels with memory has yet to be thoroughly analyzed. The conventional approach to transmission in channels with memory uses interleaving within a buffer of several codewords. However, such an approach reduces the efficiency of the redundancy embedded by the error-correcting code. It is known from information theory that considering channel memory during decoding allows the transmission rate to be increased. An evaluation of the decoding error probability of different types of low-density parity-check codes in channels with memory is presented along with estimates of minimum distance and burst error correction capability for the considered codes. The decoding error probability is estimated for conventional decoding with deinterleaving and decoding taking channel memory into account. The decoding error probability is estimated for several parameters of a channel with memory and different buffer lengths. The obtained results reveal the absence of the unique decoding approach for all parameters of the channel with memory. The best decoding error probability is determined by the degree of channel memory correlation.*

Keywords: *low-density parity-check codes, channels with memory, burst error correction, interleaving*

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Об использовании низкоплотностных кодов в канале Гилберта

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Аннотация: *Коды с низкой плотностью проверок на четность для современных стандартов связи были тщательно изучены при использовании в каналах без памяти, но исправление пакетных ошибок с их помощью не было тщательно проанализировано. В статье исследуется декодирование различных типов кодов с низкой плотностью проверок на четность в каналах с памятью и приводятся оценки* *минимального расстояния и пакетной корректирующей способности для набора низкоплотностных кодов. Рассматриваются различные сценарии декодирования для канала Гилберта, включая обычный алгоритм распространения доверия, алгоритм распространения доверия с дополнительным этапом оценки состояния канала, введение буфера с перемежением внутри буфера. Передача по каналу Гилберта сравнивается с каналом без памяти. Полученные результаты показывают, что вероятность ошибки сильно зависит от характеристик, связанных с памятью канала.*

Ключевые слова: *низкоплотностные коды, каналы с памятью, исправление пакетов ошибок, перемежение*

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Introduction

Information processing and transmission technologies are a central feature of contemporary civilized life. As such, the corruption of data as a result of the transmission system's qualities, i.e., errors occurring during information transmission, storage, or processing, represents a highly undesirable scenario. Coding theory introduces error-correction methods by adding redundancy into the data, which the receiver may then utilize to recover the original message from the corrupted data [1]. Such redundancy may be added using various error-correcting codes, e.g., low-density parity-check (LDPC) codes, which are used in many modern standards.

LDPC codes, which were invented by R. G. Gallager in 1962 [2], are used to correct errors that appear during information transmission, storage, or processing. Due to the complexity of encoding and decoding procedures and computing limitations applying at that time, LDPC codes remained largely unused for at least 30 years. However, following their rediscovery by David MacKay in the 1990s and coinciding with a breakthrough in available computing power, LDPC codes became the topic of a new wave of interest due to offering near Shannon limit error correcting capability and the development of effective encoding and decoding procedures.

Over the years, use of LDPC codes in memoryless channels piqued the interest of researchers due to their excellent error-correcting characteristics. Furthermore, LDPC codes were chosen as an error correction scheme in the proposed 5G communication standard [3]. However, since the performance of LDPC codes across channels with memory has not been properly investigated, the present work will focus on this area.

When considering mathematical models of channels, the errors that emerge during transmission are usually thought to be independent. However, in real communication channels, errors due to channel features are not independent: in this situation, the channel is said to have memory. The effect of memory in a channel may arise for several reasons, e.g., multipath propagation in fading channels [4], physical properties of storage systems, and propagation in wired channels. The existence of memory in the channel implies "unused" capacity, which encourages researchers to look for new methods for decoding dependent errors, i.e., bursts, that represent sections in transmitted sequences that may contain multiple errors, but outside of which the errors are absent or unlikely.

Although burst error correction is a well-known direction in coding theory, it is a much less researched area than independent error correction. The correction of error bursts requires the specific construction of codes (and decoding methods) aimed directly at correcting errors that appear in bursts. The typical approach to burst error correction used in modern communication standards is artificial channel decorrelation, which allows using the error-correcting code efficiently to correct independent errors. In many research papers, the estimation of decoding error probability is carried out by means of simulations, where independent errors are assumed following a binary symmetric channel (BSC) or channel with additive white Gaussian noise (AWGN). The decorrelation provided by interleaving decreases the effectiveness of the redundancy that was introduced by the error-correcting code. Another important issue is the adequateness of memoryless models to decorrelated channels with memory. In [5, 6] a method of decoding for channels with memory is described for extending the decoding procedure with an additional step. More specifically, the authors propose a channel state estimation step using the error grouping effect to lower the error probability. A comparison of random block permutation LDPC codes and Reed-Solomon codes in a Gilbert-Elliott channel is presented in [7]. In the present paper, we investigate the performance of the

LPDC codes in channels with memory based on a Gilbert model. The error correcting performance of different types of LDPC codes under conventional decoding with interleaving within the buffer is compared with decoding using knowledge about channel memory. The novelty of the paper is its investigation of decoding error probability provided by LDPC codes (including those from modern communication standards) in a Gilbert channel, considering both the known decoding methods and taking into account the specificity of the noise, as well as the decorrelation procedure for different interleaving depths, as compared to a theoretical memoryless case for infinite buffer size.

Channels with memory

When building both models of communication channels and methods of encoding and decoding the effect of noise on transmitted information is frequently represented using an additive model. The transmission can be expressed as $y = x + e$, where x is the transmitted symbol, y is the received symbol, and e is an error symbol. This formula is easily generalized to the vectors of length n as $\mathbf{y} = \mathbf{x} + \mathbf{e}$, where $\mathbf{y}$ is a received vector, $\mathbf{x}$ is a transmitted vector, $\mathbf{e}$ is an error vector. The error vector in channel models with memory comprises errors that appear in bursts.

Assuming that the channel might appear in different states during transmission and that the channel transitions from state to state over time, such a transition can be characterized in a mathematical model by appropriate transition probabilities.

Consequently, the channel can be described as a Markov chain of states, with the distinguishing feature that the Markov chain is hidden. As a consequence, there is typically no information about the state of the channel on the receiving side, implying that there is no information about the presence in a particular state of the Markov model, even with a given error vector generated by this model.

When the number of states of a communication channel is finite, the channel is called a channel with a finite number of states [8]. Such a finite-state channel model describes the states as binary symmetric channels, each having its own crossover probability. The model is frequently presumed to have only two states, namely “good” and “bad”. In a state G (good) the error probability is very low, while in state B (bad) the probability may differ depending on the channel model. Several derivations can be made using this model, such as the Gilbert model [9] (for which the error probability in the bad state is 0.5) and the Gilbert–Elliott model [10]. The Gilbert model, which is depicted in Figure 1, assumes that there are no errors in the “good” state, while errors appear with some probability in the “bad” state.

The generalization of the Gilbert model presented by E.O. Elliott in 1963 is based on the assumption that the “good” state is not error-free. The Gilbert-Elliott model describes a discrete memory channel in which the state of the channel depends on the previous state. The channel is described by two states $\Sigma = \{B, G\}$. In a “good” state, the bit error probability in the channel is P_G, while in a “bad” state P_B the probability of the channel switching from the bad to a good state is P_{BG}, while the probability of switching from good to bad is P_{GB}.

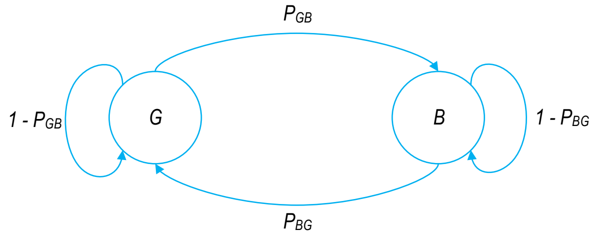

Fig. 1. Gilbert-Elliott Channel Model

LDPC codes

A binary (n, k) linear code is a k-dimensional subspace of an n-dimensional vector space over GF_2. A LDPC code can be specified by a parity-check matrix $\mathbf{H}$. If the parity-check matrix of a code is sparse, then the corresponding code is called a low-density parity-check (LDPC) code. The sparseness of $\mathbf{H}$ leads to more efficient and faster decoding because, in comparison with a dense matrix, there is a lower number of nodes to process. If the row and column weights of the parity-check matrix are constant, then the LDPC code is said to be regular; otherwise, the LDPC code is said to be irregular. Although regular codes are easier to construct, irregular codes can provide superior performance [11]. A bipartite graph (Tanner graph) is commonly used to represent the parity-check matrix of LDPC code. This graph consists of a set of check nodes and variable nodes. The minimum distance of an (n, k) linear block code C, denoted by $d_{\min}$, is defined as the smallest Hamming distance between two different codewords in C due to the linear property. Finding the minimal distance of the LDPC codes is an NP-hard problem. A probabilistic algorithm described in [12] can be used to discover words of small weight in a linear binary code. Although the work factor of the algorithm is asymptotically quite large, the method can be applied to codes of medium size. In [13], a randomized LDPC code technique that considers the properties of such codes to search for low-weight codewords is presented. In Table 1 the minimum distances for a set of LDPC codes calculated using the aforementioned algorithm are presented. In the table, n is the length of the code, k is a number of information bits, and z is a block size, d_{min} is a minimum distance, while $b_{\max}$ is a burst error correction capability.

TABLE 1. Code Parameters

Code (n, k, z)	d_{min}	b_{max}
PEG code (1008, 504)	12	218
WiFi LDPC code (1296, 648, 54)	23	53
WiMax LDPC code (1344, 672, 56)	23	55
5G LDPC code (1105, 529, 24)	16	23
Gilbert code (1000, 500, 250)	4	248
RBP LDPC code (1000, 500, 50)	-	49

Code constructions

In the present work, several types of LDPC codes are considered, including progressive-edge growth (PEG) codes [14], codes from communication standards [3], random block-permutation (RBP) codes, and Gilbert codes. Gilbert codes were initially proposed to correct error bursts. In [15], a procedure for obtaining the code error-burst correction capability based on the codes' parameters is described.

The most popular construction of LDPC codes to permit compact representation and a flexible code construction approach is a block-permutation construction. Quasi-cyclic low-density parity-check codes (QC-LDPC codes), representing a special type of the block-permutation construction, are widely used in modern communication standards due to their simple encoding implementation by means of cyclic shift registers. The class of QC-LDPC codes is described by its parity-check matrix, which consists of circulants, i.e., cyclic-permutation matrices with a cyclic shift $i_{k,j}$:

$$\mathbf{H} = \begin{bmatrix} \mathbf{C}^{i_{1,1}} & \mathbf{C}^{i_{1,2}} & \dots & \mathbf{C}^{i_{1,\rho}} \\ \mathbf{C}^{i_{2,1}} & \mathbf{C}^{i_{2,2}} & \dots & \mathbf{C}^{i_{2,\rho}} \\ \dots & \dots & \dots & \dots \\ \mathbf{C}^{i_{\gamma,1}} & \mathbf{C}^{i_{\gamma,2}} & \dots & \mathbf{C}^{i_{\gamma,\rho}} \end{bmatrix},$$

where γ – number of matrices in a column; ρ is a number of matrices in a row; $\mathbf{C}$ – $(z \times z)$-cyclic permutation matrix:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix}.$$

The QC-LDPC code is said to be regular if the matrix solely contains cyclic-permutation matrices. However, if some circulants are substituted with zero matrices, the code is called an irregular code [16]. A special case of block-permutation construction is a Gilbert code construction, whose modifications of the burst error-correcting capability b was analyzed in [15, 17]. A Gilbert code is defined by a parity-check matrix $\mathbf{H}_l$:

$$\mathbf{H}_l = \begin{bmatrix} \mathbf{I}_z & \mathbf{I}_z & \mathbf{I}_z & \dots & \mathbf{I}_z \\ \mathbf{I}_z & \mathbf{C} & \mathbf{C}^2 & \dots & \mathbf{C}^{l-1} \end{bmatrix},$$

where $\mathbf{I}_z$ – $(z \times z)$-identity matrix.

Decoding of LDPC codes

The decoding algorithms for LDPC codes are iterative procedures that operate on each symbol individually. These can be described in terms of passing messages over the edges of a Tanner graph between the check and variable nodes. The iterative nature of the algorithms refers to continuing decoding until the codeword has a zero syndrome or a predetermined number of iterations is reached. Since such iterative algorithms operate on each symbol separately, even when a large number of errors in the channel occurs and a wrong decision about the codeword is made, the bit error probability may remain low. The most common message passing algorithm with a hard decision is the bit-flipping (BF) algorithm, while the soft decision is the belief propagation (BP) algorithm, which is also known as a sum-product algorithm (SPA). Although these algorithms can provide low error probabilities, they do not guarantee error correction or burst error correction within the code error correction capability.

In [18], it was shown that, although BF cannot provide an acceptable error probability level in the case of burst error correction, it shows better results if the windowed BF is applied instead of the original BF. The BP algorithm was first presented in Gallagher's work [2]. The messages passed along the edges of this algorithm are probabilities or beliefs. However, working with log-likelihoods rather than probabilities might be useful for reducing decoding complexity. The channel log-likelihood ratios for every received value y_j for Gilbert-Elliott channel are calculated as:

$$L(y_j) = 0.5(1 - 2y_j)\log((1 - P_e)/P_e),$$

where P_e – the probability that a bit error occurs in the channel at an arbitrarily selected moment of time:

$$P_e = (P_{GB}P_B + P_{BG}P_G)/(P_{GB} + P_{BG}). \quad (1)$$

For channels with memory, it was proposed in [5] to extend the original belief propagation decoding with a channel state estimation step. In the conventional decoding algorithm, the messages are passed between the variable and check nodes of the LDPC Tanner graph. In the extended version of the algorithm, messages are passed between the nodes of the Tanner graph, between the nodes in the channel subgraph, as well as between these two subgraphs.

The decoding algorithm with an estimation step consists of an iterative message passing between the channel subgraph and code subgraph. Generally speaking, the message passing schedule may vary; in this paper, the message passing is chosen to pursue the following schedule. In the first step, incoming channel messages are sent through the channel subgraph. The messages are passed between the states of the channel subgraph. Forward messages are passed from the initial to the final state, while backward messages are passed from the final state to the initial state. The passing of messages represents the probabilities of the channel being in a specific state. Each message is a function of the previous message and the message received from the symbol node of the code graph. When the channel subgraph step is completed, the messages from the channel subgraph are passed to the code subgraph. Finally, conventional belief propagation decoding is performed.

Simulation results

In this section, the simulation results for different types of LDPC codes will be presented to evaluate the error-correcting performance using frame error rate (FER) criteria in the channel with memory. The goal of simulation is to estimate the decoding error probability per transmitted codeword in a Gilbert channel using various strategies to combat the correlated nature of the channel. On the one hand, the decoders which either use LLRs according to channel state estimation or providing single-burst correction, are considered. On the other hand, the effect of decorrelation is estimated for different buffer sizes and compared to theoretical memoryless infinite buffer case.

The general simulation scheme is presented in Figure 2. The source generates the binary data stream of length k, which is encoded by (n, k) LDPC code, obtaining an encoded binary stream of length n. Then, the buffer stores L codewords and applies interleaving within the buffer. The obtained stream is passed through the channel. Finally, the received data is deinterleaved and either fed to the channel estimator and channel decoder or passed directly to the channel decoder. In the case when buffer size is set to 1, the simulation assumes no interleaving. Thus, the considered model may be used to simulate a Gilbert channel, decorrelated Gilbert channel, or memoryless binary symmetric channel (BSC). Gilbert channel with parameters P_{GB} = 0.01 and PGB = 0.0001, P_B = 0.5 and P_G = 0, as well as the corresponding BSC channel with P_e, are calculated according to (1). The value of P_{BG} varies: low values of P_{BG} mean long rare bursts and high values mean frequent short bursts.

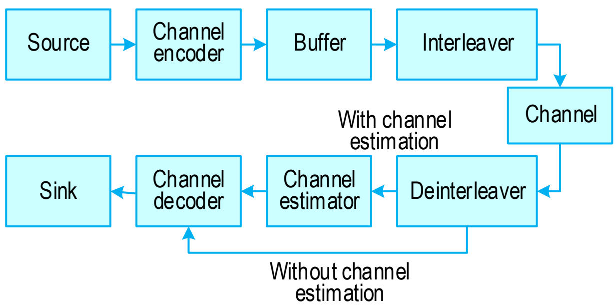

Fig. 2. Transmission Scheme

The codes and their parameters for which the simulations are obtained are presented in the Table 1. The decoding with interleaving is compared with decoding using knowledge about channel memory. The following scenarios are compared:

1) Belief propagation decoding (BP) in a Gilbert channel;

2) Belief propagation decoding with an additional channel estimation step (GE-BP) in a Gilbert channel;

3) Belief propagation decoding with interleaving inside the buffer in a Gilbert channel (the interleaving is applied inside the buffer, where L erroneous codewords are stored);

4) Belief propagation decoding in BSC channel with corresponding error probability (BP (BSC));

5) Decoding using an algorithm that is able to correct every single burst of length less than code error correction capability b_{max} in a Gilbert channel.

The simulation results for P_{GB} = 0.01, when the bursts are frequent, are presented in Figure 3. From the figures, it may be seen that GE-BP allows achieving a lower error probability due to the additional channel estimation step. It should be noted that introducing the buffer and using interleaving within the buffer permits a trade-off between FER performance and latency. The bigger the buffer, the better FER performance (gets closer to the decoding performance in the BSC channel) and the greater the latency. Both BP and GE-BP present better FER when considering lower values of P_{BG}, since the situation appears with long bursts and small gaps between bursts, which results in the big crossover probability in the corresponding BSC channel. In this case, decoding with interleaving performs worse than BP and GE-BP. The decoder that corrects every single burst of length less than code error correction capability b_{max} demonstrates unsatisfactory error correction since with P_{GB} = 0.01 and increasing P_{BG}, the bursts are short and frequent which is an unpleasant scenario for such a decoder.

Figure 4 present results for P_{GB} = 0.0001, when the bursts become rare. For small values of P_{BG} BP and GE-BP, the scheme with a buffer achieves lower values of FER comparing to the scheme with infinite buffer. Except for the last decoder that corrects single bursts, it may be seen that the curves behave similarly to the previous case with frequent bursts. Since the bursts have a sparser nature with P_{GB} = 0.0001, the decoder is able to correct a single burst within the codeword in contrast to the multiple bursts within the codeword when considering P_{GB} = 0.01.

From the simulation results, it can be seen that the range of values of the P_{BG} parameter may be divided into two areas: for small P_{BG} and high values of P_{BG}. For small P_{BG} (i.e., long error bursts), decoding considering the specifics of the presence of memory in the channel provides a gain in error probability compared to channel decorrelation. With increasing P_{BG} (with decreasing length of error bursts), the channel memory appears to a lesser extent, especially with frequent bursts, while the use of decorrelation shows a significantly better result; however, error probabilities provided by finite length buffer sizes may be far from infinite case.

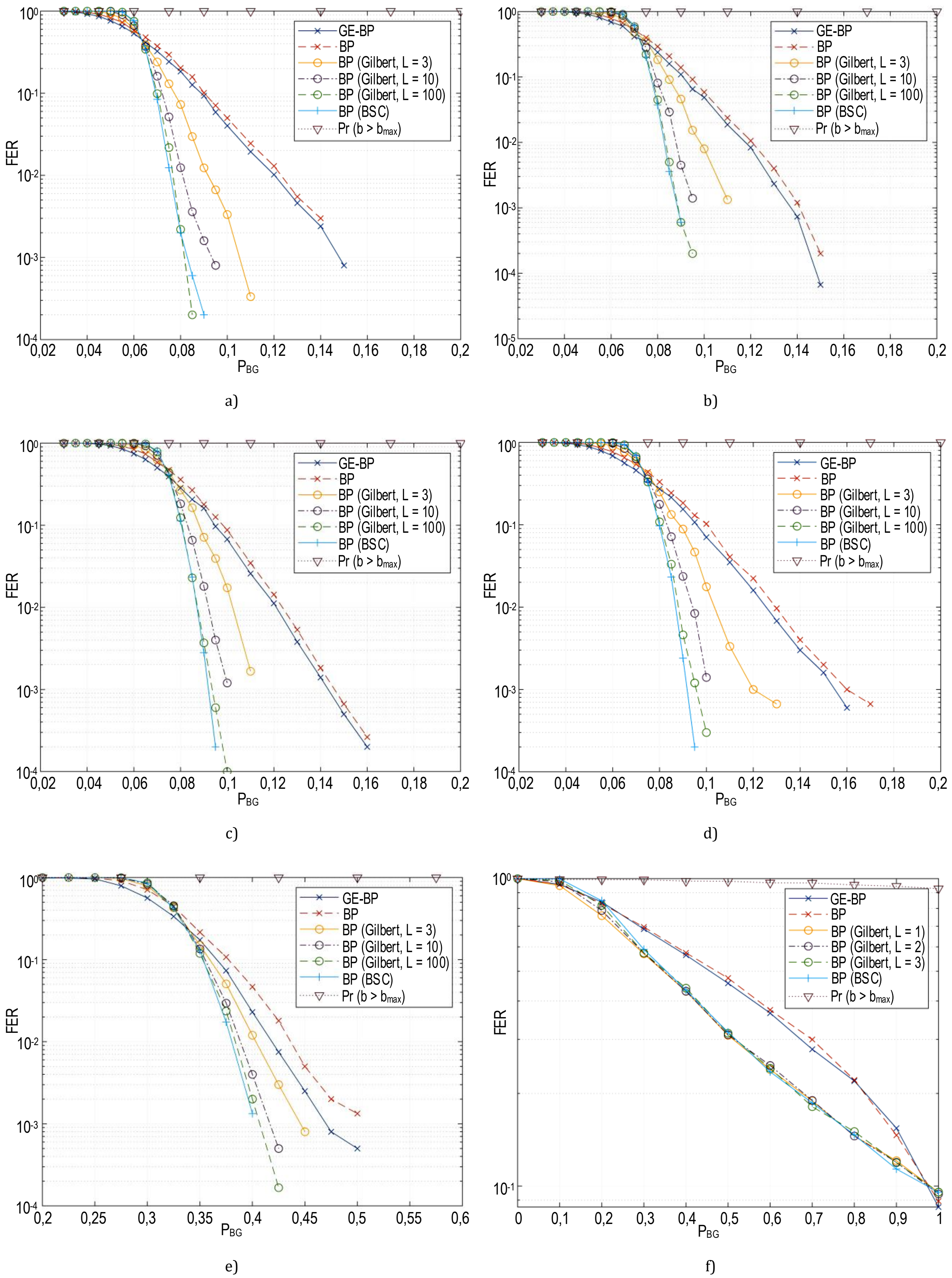

Fig. 3. FER for 5G LDPC code (a), WiFi LDPC code (b), WiMax LDPC code (c), PEG LDPC code (d), RBP LDPC code (e), Gilbert LDPC code (f) (P_{GB} = 0.01)

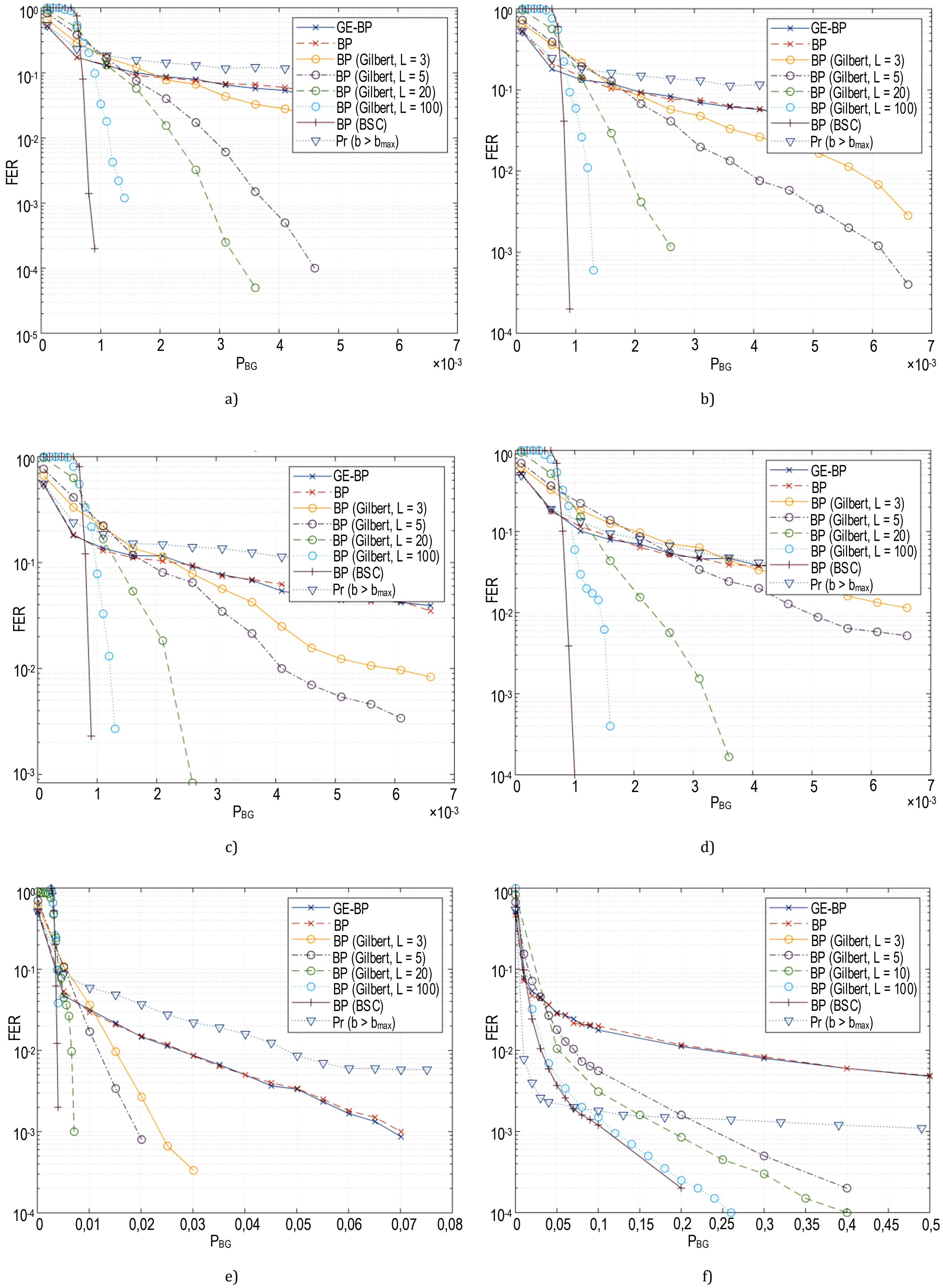

Fig. 4. FER for 5G LDPC code (a), WiFi LDPC code (b), WiMax LDPC code (c), PEG LDPC code (d), RBP LDPC code (e), Gilbert LDPC code (f) (P_{GB} = 0.0001)

Conclusion

The paper analyzes the decoding error correction performance of different types of LDPC codes in channels with memory. A conventional decoding approach with interleaving within the buffer is compared to decoding using knowledge of channel memory. The paper presents the estimations of the minimum distance and burst error correction capability of the considered codes. The investigated decoding error probability under different parameters of a channel with memory for various decoding scenarios demonstrates the existing gap between the decoding with interleaving and usage of channel memory. The specific point between the two scenarios, which represents the trade-off between the decoding performance, is determined. For P_{GB} = 0.01, this point is observed at P_{BG} = 0.07 for codes from the modern standards and at P_{BG} = 0.0007 for P_{GB} = 0.0001. Decoding using knowledge about channel memory achieves a lower decoding error probability compared to the decoding with interleaving for long bursts. However, the opposite situation, when interleaving produces lower error probability, is observed for short bursts.

Existing decoding methods that take into account the channel memory do not provide a theoretically possible gain compared to decorrelation. Thus, increasing the reliability of transmission over channels with memory is possible both in the field of constructing new codes and new methods for decoding.

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