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Statistical Arithmetic Coding Algorithm Adaptive to Correlation Properties of Wavelet Transform Coefficients

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Abstract: *It is shown that, in order to increase the compression ratio in the course of statistical arithmetic coding, it is necessary to take into account the conditional probabilities of code symbols when the preceding symbols appear. The problem of obtaining the location of the most significant symbols when encoding the current symbol is solved by calculating the autocorrelation function of the encoded symbols. An algorithm for arithmetic coding and decoding is provided, which takes into account the dependencies between the coefficients of the wavelet transform and the results of modeling its operation.*

Keywords: *image compression, arithmetic codec, adaptive arithmetic coding, wavelet transform coefficients*

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Алгоритм статистического арифметического кодирования, адаптивный к корреляционным свойствам коэффициентов вейвлет-преобразования

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Аннотация: *Обосновано, что для повышения коэффициента сжатия в ходе статистического арифметического кодирования необходимо учитывать условные вероятности при появлении предшествующих символов кода. Решена задача поиска и определения местоположения наиболее значимых символов в ходе кодирования за счет обработки результатов автокорреляционных вычислений. Приведен алгоритм арифметического кодирования и декодирования, учитывающий зависимости между коэффициентами вейвлет-преобразования и результаты моделирования его функционирования.*

Ключевые слова: *сжатие изображений, арифметический кодек, адаптивное арифметическое кодирование, коэффициенты вейвлет-преобразования*

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Introduction

Statistical arithmetic coding is a method that allows the characters of an input alphabet to be packed without loss provided that the frequency distribution of these characters is known [1, 2]. In practice, using known methods of arithmetic coding [3, 4], relative symbol frequencies are used to map a sequence of input symbols into an output data stream. Arithmetic coding is known to be optimal for achieving maximum compression on the assumption that the relative frequencies of occurrence of symbols are close to their probabilities [5]. In this case, the amount of information per character of the encoded sequence is determined by its entropy [6]. Assume that *X* is an encoded sequence of characters: $x(1), x(2)$ ..., $x(n)$, which are characterized by probabilities $P(x_1), P(x_2)$, ..., $P(x_n)$, respectively. Then the entropy of the sequence *X*:

$$H(X) = -\sum_{j=1}^{n} P(x_j)\log_2 P(x_j). \quad (1)$$

In [7], the value *H*(*X*) is considered as information contained in the sequence *S*, which determines the maximum amount of compression *S*. However, as shown in [8, 9], if only the frequencies of occurrence of symbols are considered in the encoded sequence, it is the average specific information per symbol value that is obtained, which exceeds the actual value. Let us explain this with the following reasoning.

Let the appearance of each character of the encoded data not be an independent event, i.e., the probabilities of each symbol appearing depend on the previous symbols. Therefore, if (x_1, x_2) is a complex event consisting in the sequential appearance of two characters, then the average information per character should be calculated as conditional entropy [10]:

$$H_{x_1}(x_2) = -\sum_{l,j=1}^{n} P_{S_j}(x_l)\log_2 P_{x_j}(x_l), \quad (2)$$

where $P_{x_j}(x_l)$ – is the conditional probability of the appearance of the character x_l, if it is known that the character x_j. appeared before it.

If we take into account the information about the two preceding symbols, then for the average information per one symbol, we can use the expression proposed in [7]:

$$H_{x_1x_2}(x_3) = -\sum_{l,j=1}^{n} P_{x_jx}(x_r)\log_2 P_{x_jx_l}(x_r), \quad (3)$$

where $P_{x_jx_l}(x_r)$ – conditional probability of the occurrence of the symbol x_r following the appearance of the digram x_j, x_l.

Expression (3) is easy to rewrite for *n* preceding code symbols. In this case, this expression will determine the average information per character, taking into account information about *n* previous characters. Let us denote this value as H_n. If the symbols are not independent, then, as *n* increases, the value of H_n will decrease, approaching a certain limit value H_∞, which can be considered as the theoretical value of specific information when encoding an infinitely long sequence.

Hence, the theoretical limit of the compression ratio, which follows from the above reasoning that, in order to increase the compression ratio in the course of arithmetic coding, it is necessary to take into account the conditional probabilities of code symbols when *n* previous symbols appear. At the same time, in order to achieve a compression limit close to theoretical, it is necessary that the value of *n* tend to infinity.

At the same time, when implementing real coding systems, two important limitations must be taken into account:

– the allowable delay in data transmission [11];

– the admissible complexity of encoding and decoding operations.

Both the first and second restrictions prevent a sufficiently large value of *n* being used during encoding and decoding. Then it becomes necessary to solve the problem of implementing efficient coding systems for relatively small values of *n*. In this case, a contradiction arises: on the one hand, in order to achieve the limiting compression ratios, the parameter *n* must be increased; on the other hand, for practically realizable systems, this value must be reduced. Therefore, the main aim of the present work is to describe an approach for resolving this contradiction.

Generalized statement of the problem of synthesis of algorithms for adaptive arithmetic coding, taking into account the correlation properties of the coefficients of the wavelet transform

Let the *n*+1-th character, received at the input of the arithmetic encoder, be preceded by *M* characters. Obviously, not all n preceding symbols equally carry information about the encoded symbol x_{n+1}. Then, from among the *n* preceding characters, such characters as *m* and *m*<<*n* can be chosen, which contain the most information about the encoded *n*+1-th character.

In the context of the problem to be solved, i.e., achieving the greatest compression, the term "contain the most information" refers to those m symbols by which it is possible to recover or predict the encoded symbol x_{n+1} with the greatest certainty. If wavelet transform coefficients are considered as encoded symbols, then it can be assumed that such symbols are coefficients lying in a certain neighborhood relative to the encoded coefficient and having the maximum correlation with it.

Therefore, we can hypothesize that the problem of synthesizing an arithmetic coding algorithm, which takes into account the correlation properties of the wavelet transform coefficients [12], can be reduced to solving two particular problems:

– the task of finding the most significant m coefficients from n previous ones;

– the problem of coding itself, taking into account the conditional frequencies of significant coefficients.

The problem of arithmetic coding, taking into account the conditional frequencies of encoded symbols, has already been solved [7, 8]. Therefore, we will consider the formal formulation of the first problem.

Let $\vec{V}_{1\times m}$ be defined as a vector whose values determine the locations of the most significant m symbols from the n preceding ones.

Then we can calculate the average amount of information per symbol of the code sequence in the form [13]:

$$H_{\vec{V}_{1\times m}} = -\sum_{j=1}^{n} P_{\vec{V}_{1\times m}}(S_j)\log_2 P_{\vec{V}_{1\times m}}(S_j), \quad (4)$$

where $P_{\vec{V}_{1\times m}}(S_j)$ – probability of occurrence of a character S_j subject to the appearance of an m-gram at positions determined by the vector $\vec{V}_{1\times m}$.

Further, we omit the dimension of the vector $\vec{V}_{1\times m}$.

Obviously, by changing the values of the vector $\vec{V}$, the value of $H_{\vec{V}}$ can be either decreased or increased, thereby either increasing or decreasing the compression ratio, whose limiting value is determined by the value H_∞ [14]. In this case, to increase the compression ratio of practically implemented coding systems, it is necessary to solve the problem:

$$H_{\vec{V}} \to \min_{\vec{V}}; \;\; v(i) \in \{1,2,\dots,n\}, \forall i = \overline{1,m}. \quad (5)$$

Since the values of the vector $\vec{V}$ are defined on the set of integers ranging from 1 to n, problem (5) becomes a discrete optimization problem [15]. In essence, for a given sequence of encoded symbols, it is necessary to find such a vector $\vec{V}$ for which the average amount of information per symbol of the code sequence, determined by expression (4), will be minimal.

Note that the resulting value $H_{\vec{V}}$ can never be less than the theoretical value of the specific information when encoding an infinitely long sequence but can only approach it when searching for the optimal vector $\vec{V}$, i.e., inequality $H_{\vec{V}} > H_\infty$ is always true.

Problem (5) will be considered as a generalized statement of the problem of synthesizing adaptive arithmetic coding algorithms [16], taking into account the correlation properties of the wavelet transform coefficients.

If problem (5) is solved, then the generalized block diagram of the arithmetic codec that implements the described approach can be represented as shown in Figure 1.

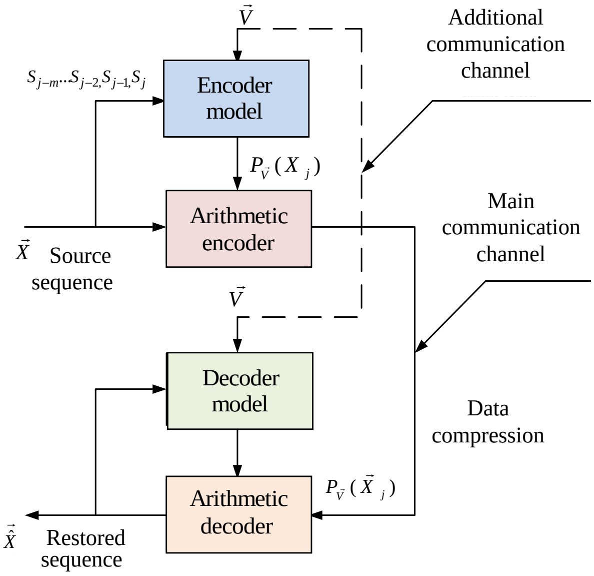

Fig 1. Generalized Block Diagram of an Arithmetic Codec That Takes into Account the Probability Conditions of the Encoded Symbols Relative to the Previous Symbols, Whose Positions are Determined by the Vector $\vec{V}$

For the operation of the arithmetic codec shown in Figure 1, a vector $\vec{V}$ minimizing $H_{\vec{V}}$ is precomputed. This vector must be transmitted to the receiving side prior to starting the actual encoding. The transmission channel of this vector is conventionally defined as an additional communication channel. Then, in the process of encoding and decoding, the arithmetic encoder and decoder, which are based on the calculation of conditional probabilities, implement the encoding and decoding operations, respectively. Conditional probabilities calculation units are defined as encoder and decoder context models.

For the practical implementation of the arithmetic codec in accordance with the block diagram shown in Figure 1, it is necessary to:

– determine the approach to solving the problem (5);

– develop algorithms for arithmetic encoding and decoding taking into account the conditional probabilities of the encoded symbols.

Below we consider the solution of these problems.

Solving the problem of finding the location of the most significant characters when encoding the current character

Consider various options for solving problem (5). As noted earlier, problem (5) is a discrete optimization problem. To solve it, either the exhaustive enumeration method or various methods of directed enumeration can be used [17]. If the vector being optimized has a relatively small dimension, then the exhaustive enumeration method can be used to solve this type of problem [18]. The essence of this method is that, for all possible solutions, the value of the objective function is calculated relative to the vector of the desired variables, allowing the solution corresponding to the extremum point to be chosen.

In the problem (5) being solved, the dimension of the desired vector is m. Moreover, each i-th element of the vector $\vec{V}$ is defined on the set from 1 to M, i.e., $v(i) \in \{1,2,\dots,n\}, \forall i = \overline{1,m}$. Then, the number of possible options is defined as the number of combinations from n to m, i.e.:

$$Var_{\vec{V}} = C_n^m. \quad (6)$$

With relatively small parameters n and m, the number of possible solutions is insignificant. So, for example, for n = 8 and m = 3, the number of solutions will be 56. However, as these parameters increase, the complexity of the problem being solved increases nonlinearly. In this case, in order to solve problem (5), it is necessary to use either classical directed enumeration methods [19] or heuristic methods of locally optimal solutions [20].

The best proven method for solving integer optimization problems is the branch and bound method [21]. However, for its implementation, the rather complicated problem arises of determining the current upper boundary of the solution at each branch point [22]. Therefore, it is proposed to use a heuristic approach to solving problem (5). This approach is based on the following assumption. As mentioned earlier, the vector $\vec{V}$ determines the positions of m characters among n preceding characters containing the most information about the encoded n + 1 character.

The essence of this assumption is that such symbols comprise coefficients lying in a certain neighborhood relative to the encoded coefficient and having the maximum correlation with it [23, 24]. Then, to calculate the vector $\vec{V}$ in problem (5), it is necessary to calculate the autocorrelation function with respect to the encoded vector. Then we determine the positions of the m maximum values of the function in a neighborhood of n symbols. Let $\vec{X} = [x(1), x(2), \dots, x(i), \dots, x(N)]$ be a vector of encoded symbols with dimension 1 × N elements. Then, the k-th value of the autocorrelation function will appear as [25, 26]:

$$R_{XX}(k) = \sum_{i=1}^{N-k} x(i) * x(i+k). \quad (7)$$

Taking into account the introduced assumption, m values of the vector $\vec{V}$ can be determined by calculating the positions of the m maximum values of function (7) for all $k \in \{1,2,\dots,n\}$.

Experiment results

In order to test the productivity of the proposed approach in the MATLAB software environment, the structure of an arithmetic codec was synthesized, with the optimal choice of the locations of the previous symbols being used in the calculation of the conditional frequencies of the encoded symbols.

Since an arbitrary combination q of the preceding symbols can be described by the column vector $B(q)$ of the matrix B. Then, when encoding the symbol $x(i)$, the q-th combination of "1" and "0" from the previous symbols, whose positions are determined by the vector $\vec{V}x(i–v(1))$, $x(i–v(2))$, ..., $x(i–v(n))$, we also define the event q.

Since the character $x(i)$ can take one of two values: $x(i)$ = 0 or $x(i)$ = 1, the following composite events may occur when encoding this character:

$$x(i) = \frac{0}{q},\ q \in \{1,\dots,2^n\}, \quad (8)$$

$$x(i) = 1/q,\ \ q \in \{1,\dots,2^n\}. \quad (9)$$

Expression (8) describes a composite event consisting in the fact that, under the condition that the event q occurs from the set of possible events {0, 1, ... , 2^n}, the symbol $x(i)$ = 0. Expression (9) describes a composite event consisting in the fact that, under the condition that the event q occurs from the set of possible events {0, 1, ... , 2^n}, the symbol $x(i)$ = 1. The events described by expressions (8) and (9) are conveniently represented by the matrix of composite events S:

$$S = \begin{Bmatrix} x(i) = 0/1 & x(i) = 0/2 & \dots & \dots & x(i) = 0/q & \dots & x(i) = 0/2^n \\ x(i) = 1/1 & x(i) = 11/2 & \dots & \dots & x(i) = 1/q & \dots & x(i) = 1/2^n \end{Bmatrix}.$$

Note that the matrix S has a dimension of 2 × 2^n elements. Each element describes a possible composite event when encoding the symbol $x(i)$. So, for example, since the element $x(i) = 0/q$ of the matrix $S(i)$, the composite event consists in the fact that $x(i)$ = 0, while the q-th binary combination is formed by n previous elements at positions determined by the vector $\vec{v}$. Each event described by the matrix $S(i)$ is associated with the conditional frequency of its occurrence, which is calculated by encoding all previous symbols of the input sequence, starting from the first and ending with the i – 1th, i.e., after encoding the i – 1-th character.

The set of all conditional occurrence frequencies of "1" and "0" after encoding the i – 1-th character, subject to the occurrence of combinations of previous characters described by the columns of the matrix B, will be represented as a matrix $P_{1,0/_B}(i-1)$:

$$P_{1,0/_B}(i-1)=\begin{cases}p_{0/_1}(i-1) & p_{0/_2}(i-1)\ldots \\ p_{1/_1}(i-1) & p_{1/_2}(i-1)\ldots\end{cases}$$

$$\left.\begin{matrix}\ldots & p_{0/_q}(i-1) & \ldots & p_{0/_{2^n}}(i-1) \\ \ldots & p_{1/_q}(i-1) & \ldots & p_{1/_{2^n}}(i-1)\end{matrix}\right\}.$$

So, for example, the elements of the q-th column of the matrix $P_{1,0/_B}(i-1)$: $p_{0/_q}(i-1)$ and $p_{1/_q}(i-1)$ are the conditional occurrence frequencies of "0" and "1", respectively, after encoding the i – 1-th character, provided that the q-th combination of previous characters occurs, as described by the q-th column of the matrix B. In order to calculate conditional frequencies $p_{0/_q}(i-1)$ and $p_{1/_q}(i-1)$, $q = 1, 2, \ldots, 2^n$, we introduce the notation:

– $N_{0/_q}(i-1)$, q=1, 2, …, 2^n – the number of "0" at the input of the encoder after encoding the i – 1-th symbol, provided that n previous elements of the q-th combination are formed;

– $N_{1/_q}(i-1)$, q=1, 2, …, 2^n – the number of "1" at the input of the encoder after encoding the i – 1-th symbol, provided that n previous elements of the q-th combination are formed. Then, by analogy with unconditional frequencies, we write the following expressions for conditional frequencies:

$$p_{0/_q}(i-1)=\frac{N_{0/_q}(i-1)}{N_{0/_q}(i-1),+N_{1/_q}(i-1)}, \qquad (10)$$

$$p_{1/_q}(i-1)=\frac{N_{1/_q}(i-1)}{N_{0/_q}(i-1),+N_{1/_q}(i-1)}, \qquad (11)$$

where $Q = 1, 2, \ldots, 2^n$.

Suppose the vector $\vec{V}=(v(1),v(2),\ldots,v(n))$, defining the positions of preceding characters, is specified. Taking into account the notation introduced above, the context model of the arithmetic encoder, which takes into account the conditional frequencies of the encoded symbols after encoding the i – 1-th symbol is determined by the following set of parameters: $N_{0/_q}(i-1)$ $N_{1/_q}(i-1), p_{0/_q}(i-1), p_{1/_q}(i-1)$, q=1, 2, …, 2^n, $L(i-1), H(i-1), R(i-1)$. Comparative data on the achievable compression ratios of the CABAC arithmetic codec (JEPEG-2000) and the developed arithmetic codec for low/medium/high frequency types of images are follow: 2,5/1,9 /1,8 vs 3,9/2,7/2,0.

The results show that for low-frequency images, the gain in compression ratio is almost 2 times. For mid-frequency and high-frequency images, the gain is from 20 % to 50 %.

Conclusion

In the course of the conducted experimental studies, it was found that the developed arithmetic codec with the optimal choice of the locations of the previous symbols involved in the calculation of the conditional frequencies of the encoded symbols has, on average, a large compression ratio as compared to classical methods of arithmetic coding. A proposed direction for further research involves the development of a block diagram of a hardware-software device for an adaptive integer arithmetic encoder that takes into account the conditional frequencies of input symbols.

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